Remarks on the existence for the one–dimensional Frémond model of shape memory alloys

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Abstract.

In this paper we outline a rigorous proof of the existence of solutions to one–dimensional initial–boundary value problems for the general and complete version of the Frémond thermo–mechanical model applying to shape memory alloys.

1. Introduction

This note is concerned with the following system of partial differential equations

$$c_0\vartheta_t - h\vartheta_{xx} = F + \partial_t\left(L\chi_1 - (\alpha(\vartheta) - \vartheta\alpha'(\vartheta))\chi_2 u_x\right) + \alpha(\vartheta)\chi_2 u_{xt}, \tag{1}$$

$$u_{tt} - \partial_x\left(-\nu u_{xxx} + \omega u_x + \alpha(\vartheta)\chi_2\right) = G, \tag{2}$$

$$k\,\partial_t \begin{bmatrix}\chi_1\\ \chi_2\end{bmatrix} + \begin{bmatrix}\ell(\vartheta-\vartheta^*)\\ \alpha(\vartheta)u_x\end{bmatrix} + \partial I_{\mathcal{K}}(\chi_1,\chi_2) \ni \begin{bmatrix}0\\ 0\end{bmatrix}, \tag{3}$$

holding in $Q = (0,1)\times(0,T)$, where $T>0$ is some final time, x and t denote space and time variables, respectively, and $\partial_t = \partial/\partial t$, $\partial_x = \partial/\partial x$. Such a system comes out from the derivation of a macroscopic model proposed by Frémond [10,11] to describe the thermo–mechanical phase transitions in shape memory materials. The equation (1) reflects the universal balance law of energy, ϑ standing for the absolute temperature, while (2) yields the equilibrium equation for the longitudinal displacement u. The relationship (3) governes the evolution of the phase proportions χ_1, χ_2 (related to the volumetric fractions of austenite and martensites phases) and it complies with the second principle of thermodynamics. As the Frémond model assumes a non–differentiable free energy (weighted sum of smooth free energies associated with the individual phases and of the mixture free energy $\vartheta I_{\mathcal{K}}$), in (3) we find the maximal monotone graph $\partial I_{\mathcal{K}}$, representing exactly the subdifferential of the indicator function $I_{\mathcal{K}}$ of the plane triangle

$$\mathcal{K} := \{(\chi_1, \chi_2) \in \mathbb{R}^2 \ : \ |\chi_2| \le \chi_1 \le 1\}$$

(convex set containing the admissible phase proportions), that is, $I_{\mathcal{K}}(\chi_1, \chi_2) = 0$ if $(\chi_1, \chi_2) \in \mathcal{K}$, $= +\infty$ otherwise. A more detailed presentation of (1–3), extending to the multidimensional case as well, is provided in [6,7] to which we refer for the physical meaning of the positive constants c_0, h, L, ν, ω, k, ℓ, and ϑ^*. Let us just point out here that the data F, G are proportional to the distributed heat source and body force, respectively, and that the function α (giving account of the thermal expansion) is non–negative, non–increasing, and vanishing above a critical temperature (the so–called Curie point) $\vartheta_c > \vartheta^*$.

Initial and boundary value problems have been investigated for various simplified versions of the field equations, in one or three dimensions of space (see [2,13,9], addressed to the one–dimensional case, and [6,1,12,3,7,4,5] quoted in chronological order), obtaining existence and, in some framework, also uniqueness and continuous dependence. Simplifications regard the removal of (part of) the nonlinearities from the energy balance equation (1) (actually, in the right hand side of (1) there are three highly nonlinear terms, namely $(\vartheta\alpha'(\vartheta) - \alpha(\vartheta))u_x\partial_t\chi_2$, $\vartheta\alpha'(\vartheta)\chi_2\partial_t u_x$, $\vartheta\alpha''(\vartheta)\chi_2 u_x\vartheta_t$, including the time derivative of phase variable or strain or temperature) and the quasi–stationary form (in which the inertial term u_{tt} is neglected) for the momentum balance equation (2). On the other hand, some effort has been done to treat the situation where $\nu = 0$, thus avoiding the regularizing fourth–order term in (2) (the use of a second gradient theory, to account for mechanical actions exerted on surfaces, is rather disputed by physicists). In addition, a possible line of future intriguing research could be the study of (1–3) with the coefficient k reduced to 0, so that no dissipation or phase relaxation enters into the dynamics of phase transition (compare with the standard multiphase Stefan problem).

However, concerning the general set of equations, in the paper [8] we have proved that, under weak and reasonable assumptions on the data, any sufficiently smooth solution has the property the absolute temperature component ϑ attains non–negative values almost everywhere. This positivity result, independent of the particular form of the momentum balance equation, plays a crucial role in the argumentation of the present paper, to show the existence of solutions to (1–3) satisfying the following boundary and initial conditions

$$h\vartheta_x(0,t) = h_0(\vartheta(0,t) - f_0(t)), \qquad -h\vartheta_x(1,t) = h_1(\vartheta(1,t) - f_1(t)), \tag{4}$$

$$u(0,t) = u(1,t) = 0, \qquad u_{xx}(0,t) = u_{xx}(1,t) = 0, \tag{5}$$

$$\vartheta(x,0) = \vartheta^0(x), \qquad u(x,0) = u^0(x), \qquad u_t(x,0) = w^0(x), \tag{6}$$

$$\chi_1(x,0) = \chi_1^0(x), \qquad \chi_2(x,0) = \chi_2^0(x), \tag{7}$$

for $t \in (0,T)$ and $x \in (0,1)$, where h_0, h_1 are positive heat exchange coefficients, the functions f_0, f_1 give the outside temperature distributions, and ϑ^0, u^0, w^0, χ_1^0, χ_2^0 denote the initial data.

In fact, our contribution is devoted to sketch the proof of the next statement. For the sake of brevity, in the notation of Sobolev spaces like $L^2(0,1)$ or $H^1(0,1)$ we omit the indication of the interval $(0,1)$. Besides, let $(\cdot\,,\cdot)$ represent both the scalar product in L^2 and the duality pairing between H^{-1} and H^1_0.

Theorem 1. *Assume that* $\vartheta^0 \in H^1$, $\vartheta^0 \geq 0$ *in* $[0,1]$, $u^0, (u^0)_{xx}, w^0 \in H_0^1$, $\chi_1^0, \chi_2^0 \in L^\infty$, $(\chi_1^0, \chi_2^0) \in \mathcal{K}$ *a.e. in* $(0,1)$, $f_0, f_1 \in W^{1,1}(0,T)$, $f_0 \geq 0$ *and* $f_1 \geq 0$ *in* $[0,T]$, $F \in L^2(0,T;L^2) \equiv L^2(Q)$, $F \geq 0$ *a.e. in* Q, $G \in W^{1,1}(0,T;L^2)$, $\alpha \in C^2(\mathbb{R})$, $\alpha'(\xi) = 0$ *if* $\xi \leq 0$ *and* $\xi \geq \vartheta_c$, *and that the constant* $c_\alpha = \|\alpha''\|_{L^\infty(\mathbb{R})}$ *is small enough (this last requirement is nothing but a compatibility condition among some data, as it will become clear in the sequel). Then the problem (1–7) has at least one solution* $(\vartheta, u, \chi_1, \chi_2)$ *with*

$$\begin{aligned}
&\vartheta \in H^1(0,T;L^2) \cap C^0([0,T];H^1) \cap L^2(0,T;H^2), \qquad \vartheta \geq 0 \quad \textit{a.e. in } Q,\\
&u \in W^{2,\infty}\left(0,T;H^{-1}\right) \cap C^1\left([0,T];L^2\right) \cap W^{1,\infty}\left(0,T;H_0^1\right) \cap C^0\left([0,T];H^2\right) \cap L^\infty\left(0,T;H^3\right),\\
&\chi_1, \chi_2 \in H^1(0,T;L^2) \cap L^\infty(Q), \qquad (\chi_1,\chi_2) \in \mathcal{K} \quad \textit{a.e. in } Q,
\end{aligned}$$

fulfilling (1) and (3) a.e. in Q, *(2) in the sense of* $L^2(0,T;H^{-1})$, *(4–5) a.e. in* $(0,T)$, *and (6–7) a.e. in* $(0,1)$.

This theorem is inferred by using a sort of elliptic regularization, deriving uniform bounds for the approximating solutions, and finally passing to the limit with the help of compactness techniques. We notice that an independent proof is proposed in [14].

2. A priori estimates

First thing, we prefer to deduce the formal a priori estimates allowing us, basically, to get the existence result. Letting the comments on approximation and limit procedure for the last section, we start by recalling that an alternative expression for (1) is

$$(c_0 - \vartheta\alpha''(\vartheta)\chi_2 u_x)\,\vartheta_t - h\vartheta_{xx} = F + L\partial_t\chi_1 + (\vartheta\alpha'(\vartheta) - \alpha(\vartheta))u_x\partial_t\chi_2 + \vartheta\alpha'(\vartheta)\chi_2 u_{xt} \quad \text{a.e. in } Q. \tag{8}$$

Moreover, a weak formulation of (2), which accounts for the boundary conditions in (5), reads

$$(u_{tt}, v) + \nu(u_{xx}, v_{xx}) + (\omega u_x + \alpha(\vartheta)\chi_2, v_x) = (G, v) \quad \forall\, v \in H_0^1 \cap H^2, \ \text{a.e. in } (0,T), \tag{9}$$

and the inclusion (3) can be equivalently rewritten as the pointwise variational inequality

$$\begin{aligned}
(\chi_1(x,t), \chi_2(x,t)) \in \mathcal{K}, \quad &\sum_{j=1}^{2} k(\partial_t\chi_j)(x,t)(\chi_j(x,t) - \gamma_j) + \ell(\vartheta(x,t) - \vartheta^*)(\chi_1(x,t) - \gamma_1)\\
&+ (\alpha(\vartheta)u_x)(x,t)(\chi_2(x,t) - \gamma_2) \leq 0 \quad \forall\, (\gamma_1,\gamma_2) \in \mathcal{K}, \qquad (10)
\end{aligned}$$

to be satisfied for a.e. $(x,t) \in Q$. By using essentially (10), the special form of the convex $\mathcal{K}$, the fact that α is constant on negative values, the sign hypotheses on F, f_0, f_1, and ϑ^0, one obtains $\vartheta \geq 0$ a.e. in Q (see [8] for the details).

The second step consists in an estimate already performed in [15] (for a different shape memory model) and involving just the energy and momentum balance equations. Indeed, we integrate (1) over $(0,1) \times (0,t)$, taking advantage of (4) and (6), and choose $v = u_t$ in (9), integrating then from 0 to $t \in [0,T]$. Summing the two identities, the terms containing $\alpha(\vartheta)\chi_2 u_{xt}$ cancel each other out. Also, owing to the properties of α and the boundedness of $\mathcal{K}$, we have that $\int_0^t\int_0^1 \partial_t\left(L\chi_1 - (\alpha(\vartheta) - \vartheta\alpha'(\vartheta))\chi_2 u_x\right) \leq 2L + 2\vartheta_c^2\, c_\alpha\left(\|u_x(\cdot,t)\|_{L^2} + \|\partial_x u^0\|_{L^2}\right)$. Hence, in view of the positivity of ϑ, by the elementary Young inequality one can easily find two constants C_1, C_2, depending only on c_0, $\|\vartheta^0\|_{L^1}$, $\|F\|_{L^1(Q)}$, h_0, h_1, $\|f_0\|_{L^1(0,T)}$, $\|f_1\|_{L^1(0,T)}$, L, ϑ_c, ω, ν, $\|\partial_x u^0\|_{H^1}$, and $\|G\|_{L^1(0,T;L^2)}$, such that

$$\|\vartheta(\cdot,t)\|_{L^1} + \|u_t(\cdot,t)\|_{L^2}^2 + \|u_x(\cdot,t)\|_{H^1}^2 \leq C_1 + C_2\, c_\alpha^2 \quad \forall\, t \in [0,T]. \tag{11}$$

Since H^1 is continuously embedded into L^∞ (here the space dimension 1 is crucial), (11) ensures that $\|u_x\|_{L^\infty(Q)} \le C_3 + C_4\, c_\alpha$ for some constants C_3, C_4 related to C_1, C_2. Now, the assumption of smallness for c_α can be made precise: in order that the coefficient of ϑ_t in (8) (such coefficient represents the specific heat which ought to) be positive eveywhere, it is demanded that $C_5 := c_0 - \vartheta_c\, c_\alpha\,(C_3 + C_4\, c_\alpha) > 0$.

The subsequent estimate gives further information about the regularity of ϑ and u as well as it deals with the phase variables χ_1, χ_2 too. Multiply formally (8) by ϑ_t, (2) by $-u_{xxt}$ (or take $v = -u_{xxt}$ in (9)), and (3) both by the vector of components $\partial_t\chi_1$, $\partial_t\chi_2$ and by the scaling constant (to be specified later) $C > 0$. Adding and integrating by parts in space and time, on account of (4–7) and of the previous bounds it is not difficult to verify that (see [5] for analogous calculations)

$$\begin{aligned} C_5 \int_0^t\!\!\int_0^1 |\vartheta_t|^2 &+ \frac{h}{2}\,\|\vartheta_x(\,\cdot\,,t)\|_{L^2}^2 + \sum_{i=0}^{1}\frac{h_i}{4}|\vartheta(i,t)|^2 + \frac{1}{2}\,\|u_{xt}(\,\cdot\,,t)\|_{L^2}^2 + \frac{\nu}{4}\,\|u_{xxx}(\,\cdot\,,t)\|_{L^2}^2 \\ +C\sum_{j=1}^{2}\frac{k}{2}\int_0^t\!\!\int_0^1 |\partial_t\chi_j|^2 \;&\le\; C_6 + C_7\int_0^t\!\!\int_0^1 \Big(|F| + \sum_{j=1}^{2}|\partial_t\chi_j| + |u_{xt}|\Big)\,|\vartheta_t| \qquad (12) \\ +\sum_{i=0}^{1} h_i \int_0^t |(f_i)_t(s)|\,|\vartheta(i,s)|ds &+ C_8\int_0^t\!\!\int_0^1 (|\vartheta_t| + |\partial_t\chi_2|)\,|u_{xxx}| + \frac{\ell\, C}{2}\int_0^t \|\vartheta(\,\cdot\,,s)\|_{L^2}^2\, ds \end{aligned}$$

for a.e. $t \in (0,T)$, where C_6, C_7, C_8 depend on the data (T included) and C_6 depends also on C. By applying the Young inequality in the right hand side of (12), we can control the integrals of $|\vartheta_t|^2$ in a way that the sum of them be less than $(C_5/2)\int_0^t\int_0^1|\vartheta_t|^2$. Then we choose C sufficiently large so that the terms containing $|\partial_t\chi_1|^2$, $|\partial_t\chi_2|^2$ are dominated by the corresponding ones in the left hand side. Finally, exploiting an extended version of the Gronwall lemma we come to the conclusion that

$$\|\vartheta\|_{H^1(0,t;L^2)\cap L^\infty(0,t;H^1)} + \|u\|_{W^{1,\infty}(0,t;H^1)\cap L^\infty(0,t;H^3)} + \sum_{j=1}^{2}\|\chi_j\|_{H^1(0,t;L^2)\cap L^\infty((0,1)\times(0,t))} \le C_9 \quad (13)$$

for all $t \in (0,T]$, C_9 being a constant with the most of dependences, according to the framework of Theorem 1.

3. Approximation

Letting $\varepsilon > 0$, we substitute (2) with the regularized equation

$$\partial_{tt}\,(u + \varepsilon u_{xxxx} - \varepsilon u_{xx}) - \partial_x\,(-\nu u_{xxx} + \omega u_x + \alpha(\vartheta)\chi_2) = G \qquad (14)$$

and we prescribe the initial datum w_ε^0 instead of w^0 (while u^0 remains unchanged), where $w_\varepsilon^0 \in H_0^1 \cap H^4$ solves the variational equality $(w_\varepsilon^0, v) + \varepsilon\,(\partial_{xx}w_\varepsilon^0, v_{xx}) + \varepsilon\,(\partial_x w_\varepsilon^0, v_x) = (w^0, v)$ for any $v \in H_0^1 \cap H^2$. Thanks to the property $w^0 \in H_0^1$, it turns out that the quantity $\|w_\varepsilon^0\|_{H^1}^2 + \sqrt{\varepsilon}\,\|\partial_{xx}w_\varepsilon^0\|_{L^2}^2 + \varepsilon\,\|\partial_{xxx}w_\varepsilon^0\|_{L^2}^2$ is bounded independently of ε, and that $w_\varepsilon^0 \to w^0$ strongly in L^∞ as $\varepsilon \searrow 0$.

Consider now the problem (1), (14), (3–7) in which w^0 is replaced by w_ε^0. For simplicity we denote this approximating problem by (P_ε). First one shows a local existence and uniqueness result for (P_ε). Namely, by applying the Contraction Mapping Principle we can find a value $\tau \in (0,T]$ (possibly depending on ε) such that, for ε sufficiently small, there exists one and only one solution of (P_ε) in the time interval $[0,\tau]$. Our fixed point argument works as follows. Take a pair $(\Theta, \mathcal{X}_2)$ with Θ, $\mathcal{X}_2 \in L^2\,(0,\tau;L^2)$ and $|\mathcal{X}_2| \le 1$ almost everywhere (see the definition of $\mathcal{K}$). Put Θ (in place of ϑ) and $\mathcal{X}_2$(in place of χ_2) in (14). Hence the initial–boundary value problem in (14), (5–6) admits a unique solution $u \in W^{2,\infty}(0,T;H^3)$. Moreover, multiplying (14) by u_t, integrating by parts in $(0,1)\times(0,t)$ $(t \le \tau)$, and observing that

$$\frac{\varepsilon}{2}\left\|\partial_{xx} w_\varepsilon^0\right\|_{L^2}^2 + \frac{\varepsilon}{2}\left\|\partial_x w_\varepsilon^0\right\|_{L^2}^2 - \int_0^t\int_0^1 \alpha(\Theta)\mathcal{X}_2 u_{xt} \le C_{10}\left(\varepsilon^{1/2} + \varepsilon^{-1/2}\int_0^t \sqrt{\varepsilon}\,\|u_{xt}(\,\cdot\,,s)\|_{L^2}\,ds\right)$$

(C_{10} independent of ε and τ), one infers that (cf. (11))

$$\|u_t(\,\cdot\,,t)\|_{L^2}^2 + \varepsilon\,\|u_{xt}(\,\cdot\,,t)\|_{H^1}^2 + \|u_x(\,\cdot\,,t)\|_{H^1}^2 \le C_1 + C_2\,c_\alpha^2 \quad \forall\, t \in [0,\tau], \tag{15}$$

provided ε and τ^2/ε are small enough. Therefore, $\|u_x\|_{L^\infty((0,1)\times(0,\tau))} \le C_3 + C_4\,c_\alpha$ (the constants are the same as in Section 2) and also $\|u_{xt}\|_{L^\infty((0,1)\times(0,\tau))}$ is bounded, by a constant proportional to $\varepsilon^{-1/2}$ (but this is not so important). Next, use the already found u_x and u_{xt} in the system coupling (1) and (3). Here you can prove the well–posedness of the corresponding initial–boundary value problem arguing as in [9], determining thus the solution $(\vartheta,\chi_1,\chi_2)$ and, in particular, a new pair (ϑ,χ_2). At this point, by means of suitable contracting estimates (similar to those developed in [9]), setting other restrictions on ε and τ if necessary, we arrange for the mapping $(\Theta,\mathcal{X}_2)\mapsto(\vartheta,\chi_2)$ to be a contraction.

Then we can proceed exactly as in the previous section, starting from the positivity of ϑ (we stress again that the result of [8] does not rely on the form of the momentum balance equation) and ending with an estimate like (13), where $\sqrt{\varepsilon}\,\|u_t\|_{L^\infty(0,t;H^3)}$ has to be added in the left hand side and where the respective constant C_9 is independent of τ and ε. Thus the solution $(\vartheta_\varepsilon,u_\varepsilon,\chi_{1\varepsilon},\chi_{2\varepsilon})$ of the problem (P_ε) actually exists in the whole interval $[0,T]$. From comparisons in (8), (4) and in (14) we recover bounds also for $\|\vartheta_\varepsilon\|_{L^2(0,T;H^2)}$ and $\|u_\varepsilon + \varepsilon\partial_{xx}(\partial_{xx}u_\varepsilon - u_\varepsilon)\|_{W^{2,\infty}(0,T;H^{-1})}$. Consequently, we are able to pass to the limit as $\varepsilon \searrow 0$ by weak and weak–star compactness and to show that any limit $(\vartheta,u,\chi_1,\chi_2)$ of subsequences of $(\vartheta_\varepsilon,u_\varepsilon,\chi_{1\varepsilon},\chi_{2\varepsilon})$ must yield one of the solutions defined by Theorem 1. In fact (see [5] for similar arguments), compact injections along with direct verifications allow us to deduce strong convergences for (subsequences of) ϑ_ε, $\partial_x u_\varepsilon$, $\chi_{1\varepsilon}$, $\chi_{2\varepsilon}$ helping to take the limit in the nonlinearities. Moreover, since $\partial_{tt}(u_\varepsilon + \varepsilon\partial_{xxxx}u_\varepsilon - \varepsilon\partial_{xx}u_\varepsilon)$ weakly–star converges to some η in $L^\infty(0,T;H^{-1})$ and $(\varepsilon\partial_{xxxx}u_\varepsilon - \varepsilon\partial_{xx}u_\varepsilon)\to 0$ in $W^{1,\infty}(0,T;H^{-1})$, it turns out that $\eta = u_{tt}$ and (9) holds.

Remark 2. In regard of experimental situations, it would be more interesting to treat the problem (1–7) with non–zero Dirichlet boundary conditions for u, assuming for instance a prescribed displacement $u(1,t) = g(t)$ on one end. In this case it suffices to let $g \in W^{3,1}(0,T)$ and use the new unknown $\widehat{u}(x,t) = u(x,t) - xg(t)$, $(x,t)\in Q$, instead of u, with obvious modifications in (1–3). What seems more difficult to handle is a Neumann boundary condition for the conormal derivative, e.g. $(-\nu u_{xxx} + \omega u_x + \alpha(\vartheta)\chi_2)(1,t) = g_n(t)$ (where g_n would represent an external traction), as it was instead done in [9] and [5], for instance. Thus, the study of (1–7) with other boundary conditions for u remains an open question as well as the extension of the above existence result to the three–dimensional case.

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